

Analysis of a recent experimental test of Bell's inequalities violating quantum predictions

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Abstract

A recent experiment by Brida et al. (quant-ph/07050439) is analyzed with the conclusion that it shows a significant violation of standard quantum predictions. A simple local hidden variables model is studied which is compatible with the empirical results and fits fairly well the deviation from the quantum predictions.

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1 Introduction

A recent experiment by Brida, Genovese and Piacentini[1] has shown a violation of an inequality which I derived[2] for a restricted, but sensible, family of local hidden variables (LHV) theories. The empirical results, however, also violate the quantum predictions. Thus it is worth studying more carefully the implications of the experiment, which is the purpose of this paper.

As is well known, many experiments have been performed in the attempt to discriminate between quantum mechanics and LHV theories via tests of Bell's inequalities. The experiments have agreed with quantum mechanics in general, but none of them has provided a conclusive, loophole-free, refutation of the whole family of LHV theories. This is due to the fact that genuine Bell inequalities, derived from locality and realism alone, are extremely difficult to test[3]. In fact the performed experiments have tested Bell type inequalities derived from local realism plus some additional assumptions. Thus the

violation of the inequalities has refuted restricted families of LHV theories, namely those fulfilling the auxiliary assumptions. In the early experiments the additional assumption was “no enhancement” [4], and the experiments provided a clear empirical refutation of that family. Later on, for about 25 years beginning with the Aspect experiments [5], the additional assumption has been “fair sampling”. LHV theories with fair sampling have been clearly refuted by many experiments. For instance the experiment by Brida et al. [1] reports a violation by 48 standard deviations (see their eq.(15)). However I think that the fair sampling assumption excludes “a priori” all sensible LHV theories [3] and therefore the refutation of those LHV theories has not too much relevance. For this reason I have started the search for Bell type inequalities derived from local realism plus some assumptions more reasonable than fair sampling, thus being able to provide tests of some restricted, but sensible, families of LHV theories.

I shall consider specifically experiments measuring polarization correlation of optical photon pairs like the one performed by Brida et al. [1]. In the experiment a source produces photon pairs, each member of the pair traveling along one of two possible paths, each path ending in an analyzer-detector system (named Alice and Bob, respectively). If the polarization planes of the analyzers are determined by the angles ϕ_1 and ϕ_2 , respectively, the results of the experiment may be summarized in two single rates, $R_1(\phi_1)$ and $R_2(\phi_2)$, and a coincidence rate $R_{12}(\phi_1, \phi_2)$. The detection rates divided by the production rate, R_0 , not measurable in the experiment, are the detection probabilities that is

$$p_j(\phi_j) = \frac{R_1(\phi_1)}{R_0}, \quad p_{12}(\phi_1, \phi_2) = \frac{R_{12}(\phi_1, \phi_2)}{R_0}, \quad (1)$$

which are the quantities to be calculated from the theory, either a LHV model or quantum mechanics. Following Bell, a LHV model consists of three functions, $\rho(\lambda)$, $P_1(\lambda, \phi_1)$, $P_2(\lambda, \phi_2)$, where λ stands for one or several hidden variables, such that the detection probabilities could be obtained by means of the integrals

$$p_j(\phi_j) = \int \rho(\lambda) P_j(\lambda, \phi_j) d\lambda, \quad p_{12}(\phi_1, \phi_2) = \int \rho(\lambda) P_1(\lambda, \phi_1) P_2(\lambda, \phi_2) d\lambda. \quad (2)$$

The essential requirements of realism and locality imply that the said func-

tions fulfil the conditions

$$\rho(\lambda) \geq 0, \int \rho(\lambda) d\lambda = 1, 0 \leq P_j(\lambda, \phi_j) \leq 1. \quad (3)$$

The experiment is compatible with local realism if there exists a LHV model reproducing the results of the experiment, that is if one may find three functions ρ, P_1, P_2 and a rate, R_0 , such that the results, R_1, R_2 and R_{12} are reproduced by eqs.(1) and (2). In particular, for the proof of compatibility it is not necessary to make any analysis of the source or the analyzer-detectors systems, which may be taken as “black boxes”. Also we should not make any assumptions about the signals produced in the source, the word “photon” being here just a short for “signal of whatever nature produced in the source and able to propagate, until its arrival to Alice or Bob, with velocity not higher than that of light”. For later convenience I shall label *LHV0* the whole family of local hidden variables theories so defined (i. e. by eqs.(1) to (3).)

The restricted family of theories which I have proposed elsewhere[2] reduces λ to a set of just two angular hidden variables, that is $\lambda \equiv \{\chi_1, \chi_2\}$, where χ_1 (χ_2) is a polarization angle of the first (second) photon of a pair, thus χ_j and $\chi_j + \pi$ representing the same polarization. In addition I assume a specific dependence of $\rho(\chi_1, \chi_2)$ and $p_j(\chi_1, \phi_j)$ so that Bell’s eqs.(2) and (3) become

$$p_{12}(\phi) = \int \rho(\chi_1 - \chi_2) P(\chi_1 - \phi_1) P(\chi_2 - \phi_2) d\chi_1 d\chi_2, \quad (4)$$

$$p_j = \int \rho(\chi_1 - \chi_2) P(\chi_j - \phi_j) d\chi_1 d\chi_2, \quad j = 1, 2, \quad (5)$$

with the conditions

$$\rho(x) = \rho(-x) \geq 0, \int \rho(x) dx = 1/\pi, 0 \leq P(x) = P(-x) \leq 1. \quad (6)$$

(The normalization of ρ fulfils eq.(3) if we integrate over both hidden variables, χ_1 and χ_2 .) I shall label *LHV1* the family defined by eqs.(4), (5) and (6).

The main inequality derived for the family *LHV1* is[2]

$$\Delta_{\text{exp}} \equiv \left\{ \frac{1}{n} \sum_{k=1}^n \left[\frac{R_{12}(\phi_k)}{\langle R_{12} \rangle} - 1 - V \cos 2\phi_k \right]^2 \right\}^{1/2} \geq D(\eta) \quad (7)$$

$$D(\eta) \simeq \frac{8\sqrt{2}}{3\pi} \sqrt{\frac{2}{3\eta} - \frac{1}{2} - \frac{\sin^4(\pi\eta/2)}{(\pi\eta/2)^4}} \varepsilon^3, \varepsilon \simeq \frac{1}{\sqrt{2}} \left(V - \frac{\sin^2(\pi\eta/2)}{(\pi\eta/2)^2} \right)_+^{1/2} \quad (8)$$

where $\phi_k = \pi k/n$, $k = 1, 2, \dots, n$, stands now for the difference between the polarization angles ϕ_1 and ϕ_2 , of Alice and Bob respectively, and $(.)_+$ means putting zero if the quantity inside the bracket is negative. The quantity η enters in the model as the ratio between twice the coincidence detection rate, $R_{12}(\phi)$, averaged over angles and the single rate $R_1 \simeq R_2$ or their mean if $R_1 \neq R_2$ (assuming that the single rates do not depend on the angles ϕ_j , as is usual) and V should be obtained from the best cosinus fit to the empirical coincidence detection rates, that is

$$\langle R_{12} \rangle = \frac{1}{n} \sum_{k=1}^n R_{12}(\phi_k), \quad \eta = \frac{4 \langle R_{12} \rangle}{R_1 + R_2}, \quad V = 2 \frac{\sum_{k=1}^n R_{12}(\phi_k) \cos 2\phi_k}{n \langle R_{12} \rangle}. \quad (9)$$

The quantity $D(\eta)$ provides a lower bound for the deviation between the best local model of the family *LHV1* and quantum mechanics, but the approximate expression eq.(8) is valid only for low detection efficiencies (see below).

According to eq.(9) the quantity η corresponds to an overall detection efficiency, taking into account all kinds of losses in lenses, polarizers, etc. But in typical experiments, including the one by Brida et al.[1], the quantity η so defined is rather small with the consequence that the inequality (7) is very well fulfilled, thus making the tests of the *LHV1* family vs. quantum mechanics almost impossible. In consequence I have proposed to restrict the family *LHV1* by including a “partial fair sampling” assumption which applies to lenses, polarizers, etc. *but not to the detectors* (see my paper[2].) This means that the quantity η to be used in the inequality (7) should be the one given by eq.(9) divided by the product $f_1 f_2 \dots f_s$, where 1, 2, ..., s correspond to the different devices inserted between the source and the detectors, like lenses, filters, polarizers or even the medium which transmits the photons, and f_i is the fraction of photons that are *not* absorbed in the corresponding device. In practice this is more or less equivalent to using for η the quantum efficiency of the detectors themselves, to be measured in auxiliary experiments.

2 Tests of local hidden variables theories vs. quantum mechanics

Now I shall study the specific experiment by Brida et al.[1]. The results of the experiment are summarized in the following table (not published in the report of the experiment[1]; I acknowledge the authors for providing me with this valuable information)

Table 1. Coincidence rates vs. angle amongst polarizers

ϕ (deg)	0	22.5	45	67.5	90	112.5	135	157.5
$R_{12}(\phi)$	9906.2	8439.6	4936.6	1454.1	108.0	1481.3	4983.5	8499.2
ΔR_{12}	21.0	18.6	13.6	9.0	8.2	11.9	14.1	19.0

Quantum mechanics predicts a cosinus curve of the form

$$\frac{R_{12}(\phi_j)}{\langle R_{12} \rangle} = 1 + V \cos(2\phi_j + \psi), \quad (10)$$

where the phase ψ is included in order to take account of any possible error in the measurement of the angle between polarizers. The best fit to the results of Table 1 gives

$$V = 0.9897, \psi = 0.31(\text{deg}). \quad (11)$$

The fit is rather bad, predicting in particular

$$R_{12}(90^\circ) = \langle R_{12} \rangle [1 + V \cos(180^\circ + \psi)] = 51.3, \quad (12)$$

which has a significant deviation from the value given in Table 1. *This shows that the quantum prediction eq.(10) is violated*, which may be also seen from the value

$$\begin{aligned} \frac{V_B}{V_A} &= 1.0205 \pm 0.0048, \\ V_A &\equiv \frac{R_{12}(0^\circ) - R_{12}(90^\circ)}{R_{12}(0^\circ) + R_{12}(90^\circ)}, V_B \equiv \sqrt{2} \frac{R_{12}(22.5^\circ) - R_{12}(67.5^\circ)}{R_{12}(22.5^\circ) + R_{12}(67.5^\circ)}, \end{aligned} \quad (13)$$

reported by Brida et al.[1]. It is clearly incompatible with the standard quantum prediction $V_A = V_B$.

If we ignore the value $R_{12}(90^\circ)$ of Table 1, a very good fit is obtained to eq.(10) with

$$V = 0.9966, \psi = 0.31(\text{deg}). \quad (14)$$

Indeed this fit reproduces all values of Table 1 well within statistical errors, except for 90° where it predicts $R_{12}(90^\circ) = 17.0$, to be compared with the value 108.0 of Table 1. We see that the violation of quantum predictions is due to the too high value of the empirical coincidence counting rate at $\phi = 90^\circ$.

In order to test the family of local models defined by eqs.(4) to (6), we must check whether the inequality (7) holds true and for this purpose we shall choose the appropriate value of the parameter η . The experiment[1] belongs to a class where half of the photons produced in the source are excluded by a post-selection procedure. Experiments of this kind have been performed since long ago[7]. However there has been some controversy about whether these experiments actually allow tests of Bell's inequalities[8]. Indeed by the nature of the source only half the photons produced belong to pairs going to different detectors (that is one to Alice and the other one to Bob) so that the effective overall detection efficiency cannot be larger than 50%, that is much lower than the minimum required for the violation of a (genuine) Bell inequality. Actually experiments of this type *do allow* Bell tests, but only if two-channel analyzers followed by two detectors are used by Alice and similarly by Bob, so that all photon pairs may (in principle) be detected[9]. In the experiment by Brida et al.[1] Alice and Bob possess only one detector each, so that it can be interpreted by a local model like the one defined by eqs.(4) and (5). Furthermore, we must use a parameter η with a value just *half the quantum efficiency of the actual detectors*, which in this experiment is quoted to be 0.62. Indeed the ratio between twice the average coincidence rate and the single rate would be half the quantum efficiency at most, the maximum taking place if there were no losses between the source and the detectors. The family of local models for the said experiment[1] with $\eta = 0.31$ will be labelled *LHV2*.

It may seem plausible to interpret the experiment by assuming that photons are particles and that the effect of the non-polarizing beam splitter is to divide the ensemble of photon pairs arriving at it (coming from the non-linear crystal) into three subensemble consisting respectively of photon pairs going: 1) both photons to Alice, 2) both to Bob, 3) one of them to Alice and the other one to Bob. Within this (corpuscular) model of light it is appro-

priate to ignore the single rates due to photons such that both members of the pair go to Alice or both to Bob, which are precisely half of the photon pairs produced in the source. Thus we may consider LHV models involving only the photon pairs of the third subensemble. If we add the “partial fair sampling” assumption, we are led to use an efficiency $\eta = 0.62$ in the inequality (7). This defines a family of models more restricted than *LHV2* which I shall label *LHV3*. Obviously we might consider also families intermediate between *LHV2* and *LHV3* or between *LHV1* and *LHV2*, each of them fulfilling inequality (7) with intermediate values of η , but I shall not discuss this possibility here.

A particular case of the family *LHV3* is obtained using a density $\rho(\chi_1 - \chi_2)$ of the form

$$\rho(x) = \frac{1}{\pi^2} [1 + (1 + \varepsilon) \cos(2x) + \varepsilon \cos(4x)], \varepsilon \in \left[0, \frac{1}{3}\right], \quad (15)$$

which was studied elsewhere[6] and I shall label *LHV4*. Thus I have defined a hierarchy of families of local models

$$LHV0 \supset LHV1 \supset LHV2 \supset LHV3 \supset LHV4. \quad (16)$$

The family *LHV1* cannot be tested without the knowledge of the single detection rates, but we may safely claim that it is not refuted by the experiment of Brida et al.. Indeed the value of η derived from eq.(9) would be very small. In contrast the family *LHV4* has been clearly refuted. In fact an inequality derived from eq.(15)[6] is violated by more than 11 standard deviations[1]. However the question whether the families *LHV2* and *LHV3* have been refuted requires a more careful analysis, which is made in the following.

Using a detection efficiency $\eta = 0.62$ for the empirical test of eq.(7) the authors[1] report a violation, by 3.3σ , of the inequality (7). However they used for $D(\eta)$ the expression eq.(8) which is valid only for relatively low efficiency. Indeed in my article[2] it is stated that eq.(8) (eq.(40) of my paper) is obtained to order ε^3 in the parameter ε (defined in my eq.(35).) In the experiment of Brida et al. $\varepsilon \simeq 0.43$ is not small and, consequently, the exact eqs.(31), (34) and (38) of the paper[2] should be used, rather than the approximation to lowest order in ε . (I apologize for not having made this point more clear in my article). Using eqs.(31) and (34) of my paper[2] we

get the following equation for ε

$$\frac{\pi - 2\varepsilon + \sin(2\varepsilon) \cos(2\varepsilon)}{\cos(2\varepsilon) [\pi - 2\varepsilon + \tan(2\varepsilon)]} = V \frac{(\pi\eta/2)^2}{\sin^2(\pi\eta/2)}, \quad (17)$$

whence, with the value eq.(11) for V and $\eta = 0.62$, I obtain $\varepsilon = 0.578$. For such a high value the calculation to lowest order in ε is not valid, but an accurate lower bound of $D(\eta)$ is obtained by means of

$$D(\eta) \geq \frac{\sqrt{2} \sin^3(2\varepsilon)}{3[(\pi - 2\varepsilon) \cos(2\varepsilon) + \sin(2\varepsilon)]} \frac{\sin^2(\pi\eta)}{(\pi\eta)^2} = 0.048. \quad (18)$$

(See eq.(39) of my paper[2]). This gives rise to a violation of the inequality (7) even stronger than the one reported[1] so that the experiment clearly refutes the family *LHV3*.

As said above an effective efficiency $\eta = 0.31$ should be used in the test of the family *LHV2* via inequality (7). In this case we may take $D(\eta)$ as given by eq.(8) because the parameter ε has the value $\varepsilon = 0.1820$, which is low enough for the approximations involved being valid (the exact eq.(17) gives $\varepsilon = 0.1825$). Thus I get from eq.(8)

$$D(\eta) = 0.0065 < \Delta_{\text{exp}} = 0.0074,$$

that is the inequality (7) of the family *LHV2* is fulfilled. (The lower bound eq.(18) is now $D(\eta) \geq 0.0052$.)

The model of the family *LHV2* which is most close to quantum mechanics predicts a deviation from the best cosinus fit (that is eq.(10) with $V = 0.9897$) of the form (see eq.(37) of my paper[2])

$$\begin{aligned} \delta(\phi) &= \alpha [\beta \cos(2\phi) - 1] + \gamma(\phi), \\ \alpha &\equiv \frac{8\varepsilon^3}{3\pi}, \beta \equiv 2 \frac{\sin^2(\pi\eta/2)}{(\pi\eta/2)^2}, \gamma(\phi) = \frac{2\alpha}{\eta^2} \left(\eta + \frac{2}{\pi} |\phi| - 1 \right)_+, \end{aligned} \quad (19)$$

where ε was defined in eq.(8), $\phi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $()_+$ means putting 0 if the quantity inside brackets is negative. With $\eta = 0.31$ the first two terms of eq.(19) predict an effective increase in the parameter V leading to

$$V \rightarrow V_{\text{eff}} = \frac{V + \alpha\beta}{1 - \alpha} \simeq V + \alpha(1 + \beta) = 1.003$$

which is somewhat larger than V in the best cosinus fit to the data of Table 1 when the value $R_{12}(90^\circ)$ is excluded, see eq.(14). The contribution of the last term of eq.(19) is either zero or negligible for all angles reported in Table 1 except $\phi = 90^\circ$ (note that any angle $\phi > 90^\circ$ in Table 1 should be replaced by $180^\circ - \phi$ if used in eq.(19).) For 90° we get $\gamma(90^\circ) = 0.0330$, $\delta(90^\circ) = 0.0184$ which, taking into account the value $\langle R_{12} \rangle \simeq 4980$ obtained from Table 1, gives a predicted increase $\Delta R_{12}(90^\circ) = 89.6$. If this is added to the prediction of the fit eq.(11), we get a LHV model prediction $R_{12}(90^\circ) \simeq 140.9$, which is somewhat larger than the empirical datum of Table 1. The results of our calculation strongly suggest that a local model defined by eqs.(4) to (6), with a value of the parameter η slightly smaller than 0.31, may agree with the empirical results. But this point will not be studied further in the present paper.

3 Conclusions

In summary the results of the experiment by Brida et al., shown in Table 1, are not compatible with the standard predictions of quantum mechanics, eq.(10). Nevertheless it may be that small corrections, not included in the standard quantum calculations, might account for the disagreement between theory and experiment. In contrast a sensible family of local models[2] predicts fairly well the empirical departure from standard quantum mechanics, that is a substantial increase in the coincidence counting rate at angles close to $\phi = 90^\circ$.

I will finish stating that the experiment by Brida et al.[1] is remarkable in that it has achieved, for the first time to my knowledge, a value of the parameter V_B very close to unity (the departure is only 1.5 per thousand) combined with a fairly high quantum efficiency of the detectors. These properties are crucial for the discrimination between standard quantum predictions and sensible families of local hidden variables theories, like the one defined by eqs.(4) to (6).

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